Precision tests of Standard Model with leptonic and semileptonic kaon decays

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ABSTRACT: Till the middle of 2004 it appeared that the unitarity relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ might not hold at the 2.3σ level. At that time, however, $|V_{us}|$ was inferred from old experimental data. Since then, a large experimental and theoretical effort has been invested leading to a removal of the problem. Thanks to the new and improved measurements by BNL-E865, KLOE, KTeV, ISTRA+ and NA48, the $K_{\ell 3}$ decay rate moved up so that $|V_{us}|$ is now consistent with unitarity. On the theory side, much progress has been made in order to tame the systematic uncertainties related to the computation of the $K_{\ell 3}$ form factors.

This joint progress allowed to assess the validity of the CKM unitarity relation at the level of less than 1%. Recent measurements of kaon decays contributing to the determination of $|V_{us}|$ $|V_{us}|/|V_{ud}|$ are summarized, and up-to-date evaluations of $|V_{us}|f_+(0)$ and $|V_{us}|$ are presented. In addition, we discuss the sensitivity of $K_{\ell 3}$ and $K_{\ell 2}$ decays to various scenarios of physics beyond Standard Model.

KEYWORDS: Vus, CKM, kaon.

*most of this material is done in conjunction with [1]

Contents

1. Introduction

I report on precise tests of the Standard Model (SM) with kaon decays using world data. From the experimental information on down- to up-quark transitions (such as $d \to u$, $s \to u$ and $b \to u$), we access the effective dimesion-six operators of the form, $\overline{D}\,\Gamma_1 U \bar{\ell}\,\Gamma_2\,\nu$, with D (U) being a generic "*down*" ("*up*") flavor, and $\ell = e$, μ, τ. Their effective coupling are parametrized as the SM contribution $G_F^2\,|V_{UD}|^2$, plus a possible new physics terms, $G_F^2\,\epsilon_{NP}$. Since the dimension-six operators are not protected by gauge invariance

the possible effects of non-decoupling are proportional to $(1 + M_W^2/\Lambda_{NP}^2)$. The effects of these non-standard contributions cannot be very large, but are possibly detectable in high-precision experiments.

A convenient strategy to measure these effects against the SM parameters, G_F^2 and $|V_{UD}|$, rely on the Cabibbo universality hypothesis (or unitarity constraint):

$$G_{CKM}^2 = G_\mu^2 \ \left(\text{or } |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \text{ and } G_F \equiv G_\mu\right), \tag{1.1}$$

where $G_{CKM}^2 = G_F^2 \left(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right)$, and $G_\mu = 1.166371(6) \times 10^{-5}\text{GeV}^{-2}$, as extracted from the accurate measurement of the muon lifetime [2].

I report on the progress related to the verification of the unitarity relation (1.1). As we shall see the current accuracy of the CKM unitarity relation (1.1), is at the 0.1% level, becoming an important constraint to the model builders of scenarios beyond SM physics. $K_{\ell 3}$ and $K_{\mu 2}$ decays offer possibly the cleanest way to test us transitions. ud transitions are precisely measured in superallowed nuclear β-decays. The most recent determination of V_{ud} is $|V_{ud}|$=0.97418 $\pm$ 0.00026, [3].

This report is organized as follows. The phenomenological farmework needed to describe $K_{\ell 3}$ and $K_{\mu 2}$ decays is briefly recapitulated in Section 1.1. Section 2 is dedicated to the combination of the experimental data. The results and interpretations are presented in Section 3.

1.1 $K_{\ell 2}$ and $K_{\ell 3}$ phenomenology

For $K_{\ell 2}$ ($\pi_{\ell 2}$) amplitudes, we introduce the following QCD parameters

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 u|K^+(p)\rangle = i\sqrt{2} f_K\, p_\mu, \quad \langle 0|\bar{s}\gamma_5 u|K^+\rangle = -i\sqrt{2} f_K \frac{m_K^2}{m_s + m_u}\ , \tag{1.2}$$

For $K_{\ell 3}$ amplitudes, we define the following form factors

$$\begin{aligned} \langle \pi^+(k)\,|\bar{s}\gamma^\mu u|K^0(p)\rangle &= \frac{1}{\sqrt{2}}\Big((p+k)^\mu f_+^{K^0\pi^+}(t) + (p-k)^\mu f_-^{K^0\pi^+}(t)\Big) \\ f_-^{K\pi}(t) &= \frac{m_K^2 - m_\pi^2}{t}\left(f_0^{K\pi}(t) - f_+^{K\pi}(t)\right) \end{aligned} \tag{1.3}$$

where $t = (p-k)^2$ and

$$\langle \pi^+(k)\,|\bar{s}u|K^0(p)\rangle = -\frac{M_K^2 - M_\pi^2}{\sqrt{2}\,(m_s - m_u)} f_0(t) \tag{1.4}$$

The SM gives the following relations for the $K_{\ell 3}$ and $K_{\ell 2}$ decay rates:

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{G_\mu^2 M_K^5}{192\pi^3} C_K S_{\text{ew}}\,|V_{us}|^2 f_+(0)^2\, I_K^\ell(\lambda_{+,0})\,\left(1 + \delta_{SU(2)}^K + \delta_{\text{em}}^{K\ell}\right)^2 \quad , \tag{1.5}$$

$$\frac{\Gamma(K^\pm_{\ell 2(\gamma)})}{\Gamma(\pi^\pm_{\ell 2(\gamma)})} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi}\left(\frac{1 - m_\ell^2/m_K^2}{1 - m_\ell^2/m_\pi^2}\right) \times (1 + \delta_{\text{em}}) \quad , \tag{1.6}$$

where $C_K = 1$ (1/2) for the neutral (charged) kaon decay. $I_K^\ell(\lambda_{+,0})$ is the phase space integral which also includes the form factors parameterized by $\lambda_{+,0}$. The universal short-distance electromagnetic correction, $S_{\text{ew}} = 1.0232(3)$, has been computed at $\mu = M_\rho$ in

ref. [4], while the long-distance electromagnetic corrections, $\delta_{\rm em} = 0.9930(35)$[5] and $\delta_{\rm em}^{K\ell}$, as well as the isospin-breaking ones, $\delta_{SU(2)}^{K}$, have been recently revised in ref. [6] (see table 1).

	$\delta_{SU(2)}^{K}(\%)$	$\delta_{\rm em}^{K\ell}(\%)$
K_{e3}^0	0	+0.57(15
K_{e3}^+	2.36(22)	+0.08(15)
$K_{\mu3}^0$	0	+0.80(15)
$K_{\mu3}^+$	2.36(22)	+0.05(15)

Table 1: Summary of the isospin-breaking factors [7, 6]

with correlations for $\delta_{\rm em}^{K\ell}(\%)$:

$$\begin{pmatrix} 1. & 0.11 & 0.78 & -0.12 \\ & 1. & -0.12 & 0.78 \\ & & 1. & 0.11 \\ & & & 1. \end{pmatrix} \tag{1.7}$$

The remaining quantities, $f_+(0)$, the vector form factor at zero momentum transfer $[q^2 = (p_K - p_\pi)^2 = 0]$, and f_K/f_π, the ratio of the kaon and pion decay constants contains the non-perturbative QCD information on the flavor SU(3) breaking effects arising in the relevant hadronic matrix element. For more details see Sec. 3.2.1.

1.2 Form Factors Parameterizations: Lattice QCD, ChPt and Dispersion relations

To determine V_{us}, we have to determine the integral over phase space of the Dalitz density, which depends on the form of $f_{+,0}(t)$, the form factors (FF) in eq. (1.3). To reduce uncertainties, it would be convenient to have theoretical information on their t dependence. Our theorical knowledge is still poor. ChPt and Lattice QCD could be very useful but predictivity of ChPt is limited by our knowledge of Low Energy Constants(LECs), while lattice calculations still have large uncertainty.

For this reason, present estimates of the phase space integral mainly rely on measurements. In the physical region, it is a good approximation to use a quadratic t dependence of the FFs such as

$$\tilde{f}_{+,0}(t) \equiv \frac{f_{+,0}(t)}{f_{+,0}(0)} = 1 + \lambda'_{+,0}\, \frac{t}{m^2} + \frac{1}{2}\, \lambda''_{+,0} \left(\frac{t}{m^2}\right)^2 \tag{1.8}$$

Note that $t = (p_K - p_\pi)^2 = m_K^2 + m_\pi^2 - 2m_K E_\pi$, therefore the FFs depends only on E_π. The FF parameters can thus be obtained from a fit to the pion spectrum which is of the form $g(E_\pi) \times \tilde{f}(E_\pi)^2$. Unfortunately t is maximum for E_π=0, where $g(E_\pi)$ vanishes. Still, experimental information about $\tilde{f}_+$ are quite accurate and a pole parametrization, $\lambda''_+ \sim 2(\lambda'_+)^2$, looks confirmed from present data (see later). Theory would also support this hypothesis. The situation of $\tilde{f}_0$ instead is completely open. The main problem is that λ''_0 cannot be detected from the data and we can not discriminate among different assumptions

such as linear, pole or quadratic fits. In turn, these model ambiguities induce a systematics uncertainty for $|V_{us}|$ even though data for partial rates by itself are very accurate. For this reason, hints from theory are welcome and it is advisable to use model-independent parametrisations as possible.

The vector and scalar form factors $f_{+/0}(t)$ in eq. (1.3) are analytic functions in the complex t–plane, except for a cut along the positive real axis, starting at the first physical threshold $t_{th\,1} = (M_K + M_\pi)^2$, where their imaginary parts develop discontinuities. They are real for $s < t_{th\,1}$.

Cauchy's theorem implies that $f_{+/0}(t)$ can be written as a dispersive integral along the physical cut

$$f(t) \;=\; \frac{1}{\pi} \int_{t_{th\,1}}^{\infty} ds' \, \frac{\mathrm{Im} f(s')}{(s' - t - i0)} + \text{subtractions} \,. \tag{1.9}$$

where all possible on-shell intermediate states contribute. A number of subtractions are needed to make the integral convergent. Particularly appealing is an improved dispersion relation recently proposed in ref. [8], which reads

$$\tilde{f}_0(t) = \left(\tilde{f}_0(t_{CT})\, e^{\left(\frac{t}{t_{\mathrm{CT}}} - 1\right) H(t)} \right)^{\frac{t}{t_{\mathrm{CT}}}} \,, \quad H(t) = \frac{t_{\mathrm{CT}}^2}{\pi} \int_{t_{\mathrm{CT}}}^{\infty} \frac{dx}{x} \frac{\phi(x)}{(x - t_{\mathrm{CT}})\,(x - t - i\epsilon)} \tag{1.10}$$

Here $\phi(x)$, the phase of $f_0(t)$, is taken from $K\pi$ scattering. This dispersive form has been solved in terms of the subtractions at $t = 0$ (where by default, $\tilde{f}_0(0) \equiv 1$ is known) and at $t_{CT} = (m_K^2 - m_\pi^2)$, where the QCD soft-pion theorem, known as Callan-Treiman relation, implies

$$\tilde{f}_0(t \equiv (m_K^2 - m_\pi^2)) = \frac{f_K}{f_\pi} \frac{1}{f_+(0)} + \; (3.5 \pm 8.0\, 10^{-3}[9]) \tag{1.11}$$

By exploiting this form on the spectrum of $K_{\ell 3}$ data, we have to estimate one unknown parameter ($\tilde{f}_0(t \equiv (m_K - m_\pi)^2)$ or $f_K/f_\pi/f_+(0)$), whereas, eq. (1.10) within its theoretical uncertainty non-trivially contraints the coefficients of the Taylor's expansion in eq. (1.8)(for axample for the first two derivatives, we have from ref. [8] $\lambda_0'' = (\lambda_0')^2 - 2m_\pi^4/t_{CT} G'(0) = (\lambda_0')^2 + (4.16 \pm 0.50) \times 10^{-4}$). By the experiemtal information of $\tilde{f}_0(t \equiv (m_K - m_\pi)^2)$, Callan-Treiman theorem allows for a consistency checks with lattice QCD, where $f_K/f_\pi/f_+(0)$ can be estimated (see section3.2.3).

2. Data Analysis

We perform fits to world data on the BRs and lifetimes for the K_L and $K^\pm$, with the constraint that BRs add to unity[1]. This is the most satisfactory way of making use of the new measurements.

2.1 K_L leading branching ratios and τ_L

Numerous measurements of the principal K_L BRs, or of various ratios of these BRs, have been published recently. For the purposes of evaluating $|V_{us}| f_+(0)$, these data can be used in a PDG-like fit to the K_L BRs and lifetime, so all such measurements are of interest.

KTeV has measured five ratios of the six main K_L BRs [10]. The six channels involved account for more than 99.9% of the K_L width and KTeV combines the five measured ratios to extract the six BRs. We use the five measured ratios in our analysis: $\mathrm{BR}(K_{\mu3}/K_{e3}) = 0.6640(26)$, $\mathrm{BR}(\pi^+\pi^-\pi^0/K_{e3}) = 0.3078(18)$, $\mathrm{BR}(\pi^+\pi^-/K_{e3}) = 0.004856(28)$, $\mathrm{BR}(3\pi^0/K_{e3}) = 0.4782(55)$, and $\mathrm{BR}(2\pi^0/3\pi^0) = 0.004446(25)$. The errors on these measurements are correlated; this is taken into account in our fit.

NA48 has measured the ratio of the BR for K_{e3} decays to the sum of BRs for all decays to two tracks, giving $\mathrm{BR}(K_{e3})/(1 - \mathrm{BR}(3\pi^0)) = 0.4978(35)$ [11]. From a separate measurement of $\mathrm{BR}(K_L \to 3\pi^0)/\mathrm{BR}(K_S \to 2\pi^0)$, NA48 obtains $\mathrm{BR}(3\pi^0)/\tau_L = 3.795(58)$ $\mu\mathrm{s}^{-1}$ [12].

Using $\phi \to K_L K_S$ decays in which the K_S decays to $\pi^+\pi^-$, providing normalization, KLOE has directly measured the BRs for the four main K_L decay channels [14]. The errors on the KLOE BR values are dominated by the uncertainty on the K_L lifetime τ_L; since the dependence of the geometrical efficiency on τ_L is known, KLOE can solve for τ_L by imposing $\sum_x \mathrm{BR}(K_L \to x) = 1$ (using previous averages for the minor BRs), thereby greatly reducing the uncertainties on the BR values obtained. Our fit makes use of the KLOE BR values before application of this constraint: $\mathrm{BR}(K_{e3}) = 0.4049(21)$, $\mathrm{BR}(K_{\mu3}) = 0.2726(16)$, $\mathrm{BR}(K_{e3}) = 0.2018(24)$, and $\mathrm{BR}(K_{e3}) = 0.1276(15)$. The dependence of these values on τ_L and the correlations between the errors are taken into account. KLOE has also measured τ_L directly, by fitting the proper decay time distribution for $K_L \to 3\pi^0$ events, for which the reconstruction efficiency is high and uniform over a fiducial volume of $\sim 0.4\lambda_L$. They obtain $\tau_L = 50.92(30)$ ns [13].

There are also two recent measurements of $\mathrm{BR}(\pi^+\pi^-/K_{\ell3})$, in addition to the KTeV measurement of $\mathrm{BR}(\pi^+\pi^-/K_{e3})$ discussed above. KLOE obtains $\mathrm{BR}(\pi^+\pi^-/K_{\mu3}) = 7.275(68) \times 10^{-3}$ [15], while NA48 obtains $\mathrm{BR}(\pi^+\pi^-/K_{e3}) = 4.826(27) \times 10^{-3}$ [16]. All measurements are fully inclusive of inner bremsstrahlung. The KLOE measurement is fully inclusive of the direct-emission (DE) component, DE contributes negligibly to the KTeV measurement, and a residual DE contribution of 0.19% has been subtracted from the NA48 value to obtain the number quoted above. For consistency, in our fit, a DE contribution of 1.52(7)% is added to the KTeV and NA48 values. Our fit result for $\mathrm{BR}(\pi^+\pi^-)$ is then understood to be DE inclusive.

In addition to the 14 recent measurements listed above, our fit for the seven largest K_L BRs and lifetime uses four of the remaining five inputs to the 2006 PDG fit and the constraint that the seven BRs add to unity. The results are given in Table 2.

The evolution of the average values of the BRs for $K_{L\ell3}$ decays and for the important normalization channels is shown in Fig. 2.

Our fit gives $\chi^2/\mathrm{ndf} = 20.2/11$ (4.3%), while the 2006 PDG fit gives $\chi^2/\mathrm{ndf} = 14.8/10$ (14.0%). The differences between the output values from our fit and the 2006 PDG fit are minor. The poorer value of χ^2/ndf for our fit can be traced to contrast between the KLOE value for $\mathrm{BR}(3\pi^0)$ and the other inputs involving $\mathrm{BR}(3\pi^0)$ and $\mathrm{BR}(\pi^0\pi^0)$—in particular, the PDG ETAFIT value for $\mathrm{BR}(\pi^0\pi^0/\pi^+\pi^-)$. The treatment of the correlated KTeV and KLOE measurements in the 2006 PDG fit gives rise to large scale factors for $\mathrm{BR}(K_{e3})$ and $\mathrm{BR}(3\pi^0)$; in our fit, the scale factors are more uniform. As a result, our value for $\mathrm{BR}(K_{e3})$

Parameter	Value	S
BR(K_{e3})	0.40563(74)	1.1
BR($K_{\mu 3}$)	0.27047(71)	1.1
BR($3\pi^0$)	0.19507(86)	1.2
BR($\pi^+\pi^-\pi^0$)	0.12542(57)	1.1
BR($\pi^+\pi^-$)	$1.9966(67) \times 10^{-3}$	1.1
BR($2\pi^0$)	$8.644(42) \times 10^{-4}$	1.3
BR($\gamma\gamma$)	$5.470(40) \times 10^{-4}$	1.1
τ_L	51.173(200) ns	1.1

Table 2: Results of fit to K_L BRs and lifetime

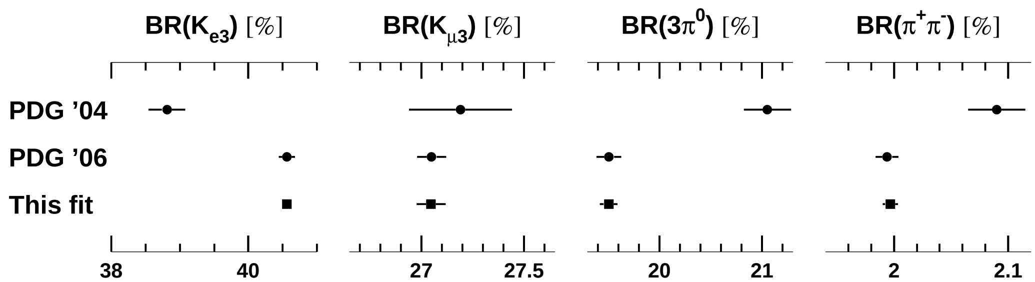

Figure 1: Evolution of average values for main K_L BRs.

has a significantly smaller uncertainty than does the 2006 PDG value.

2.2 K_S leading branching ratios and τ_S

KLOE has published [17] a measurement of BR($K_S \to \pi e \nu$) that is precise enough to contribute meaningfully to the evaluation of $|V_{us}| f_+(0)$. The quantity directly measured is BR($\pi e \nu$)/BR($\pi^+\pi^-$). Together with the published KLOE value BR($\pi^+\pi^-$)/BR($\pi^0\pi^0$) = 2.2459(54), the constraint that the K_S BRs must add to unity, and the assumption of universal lepton couplings, this completely determines the K_S BRs for $\pi^+\pi^-$, $\pi^0\pi^0$, K_{e3}, and $K_{\mu 3}$ decays [18]. In particular, BR($K_S \to \pi e \nu$) = $7.046(91) \times 10^{-4}$.

NA48 has recently measured the ratio $\Gamma(K_S \to \pi e \nu)/\Gamma(K_L \to \pi e \nu) = 0.993(26)(22)$ [19]. The best way to include this measurement in our analysis would be via a combined fit to K_S and K_L branching ratio and lifetime measurements. Indeed, such a fit would be useful in properly accounting for correlations between K_S and K_L modes introduced with the preliminary NA48 measurement of $\Gamma(K_L \to 3\pi^0)$, and more importantly, via the PDG ETAFIT result, which we use in the fit to K_L branching ratios. At the moment, however, we fit K_S and K_L data separately. NA48 quotes BR($K_S \to \pi e \nu$) = $7.046(180)(160) \times 10^{-4}$; averaging this with the KLOE result gives BR($K_S \to \pi e \nu$) = $7.046(84) \times 10^{-4}$, improving the accuracy on this BR by about 10%.

For τ_{K_S} we use 0.8958×10^{-10} s, where this is the non-CPT constrained fit value from the PDG, and is dominated by the 2002 NA48 and 2003 KTeV measurements.

Parameter	Value	S
$\mathrm{BR}(K_{\mu 2})$	63.569(113)%	1.1
$\mathrm{BR}(\pi\pi^0)$	20.644(80)%	1.1
$\mathrm{BR}(\pi\pi\pi)$	5.5953(308)%	1.0
$\mathrm{BR}(K_{e3})$	5.0780(258)%	1.2
$\mathrm{BR}(K_{\mu 3})$	3.3650(271)%	1.7
$\mathrm{BR}(\pi\pi^0\pi^0)$	1.7495(261)%	1.1
$\tau_\pm$	12.3840(193) ns	1.7

Table 3: Results of fit to $K^\pm$ BRs and lifetime

2.3 $K^\pm$ leading branching ratios and $\tau^\pm$

There are several new results providing information on $K^\pm_{\ell 3}$ rates. These results are mostly preliminary and have not been included in previous averages.

NA48/2 has recently published measurements of the three ratios $\mathrm{BR}(K_{e3}/\pi\pi^0)$, $\mathrm{BR}(K_{\mu 3}/\pi\pi^0)$, and $\mathrm{BR}(K_{\mu 3}/K_{e3})$ [20]. These measurements are not independent; in our fit, we use the values $\mathrm{BR}(K_{e3}/\pi\pi^0) = 0.2470(10)$ and $\mathrm{BR}(K_{\mu 3}/\pi\pi^0) = 0.1637(7)$ and take their correlation into account. ISTRA+ has also updated its preliminary value for $\mathrm{BR}(K_{e3}/\pi\pi^0)$. They now quote $\mathrm{BR}(K_{e3}/\pi\pi^0) = 0.2449(16)$[21].

KLOE has measured the absolute BRs for the K_{e3} and $K_{\mu 3}$ decays [22] and a very precise measuremet of $\mathrm{BR}(K_{\mu 2})$[23]. In $\phi \to K^+K^-$ events, K^+ decays into $\mu\nu$ or $\pi\pi^0$ are used to tag a K^- beam, and vice versa. KLOE performs four separate measurements for each $K_{\ell 3}$ BR, corresponding to the different combinations of kaon charge and tagging decay. The final averages are $\mathrm{BR}(K_{e3}) = 4.965(53)\%$ and $\mathrm{BR}(K_{\mu 3}) = 3.233(39)\%$.

Very recently KLOE has also measured the absolute branching ratio for the $\pi\pi^0$ decay with 0.5% accuracy. The KLOE preliminary result, is $\mathrm{BR}(\pi\pi^0) = 0.20658(112)$[24].

Our fit takes into account the correlation between these values, as well as their dependence on the $K^\pm$ lifetime. The world average value for $\tau_\pm$ is nominally quite precise; the 2006 PDG quotes $\tau_\pm = 12.385(25)$ ns. However, the error is scaled by 2.1; the confidence level for the average is 0.17%. It is important to confirm the value of $\tau_\pm$. The two new measurements from KLOE, $\tau_\pm = 12.364(31)(31)$ ns and $\tau_\pm = 12.337(30)(20)$ ns[25] with correlation 34%, agree with the PDG average.

Our fit for the six largest $K^\pm$ BRs and lifetime makes use of the results cited above, plus the data used in the 2006 PDG fit, except for the Chiang '72 measurements for a total of 26 measurements. The six BRs are constrained to add to unity. The results are shown in Table 3.

The fit quality is poor, with $\chi^2/\mathrm{ndf} = 42/20$ (0.31%). However, when the five older measurements of $\tau_\pm$ are replaced by their PDG average with scaled error, χ^2/ndf improves to 24.3/16 (8.4%), with no significant changes in the results.

Both the significant evolution of the average values of the $K_{\ell 3}$ BRs and the effect of the correlations with $\mathrm{BR}(\pi\pi^0)$ are evident in Fig. 2.

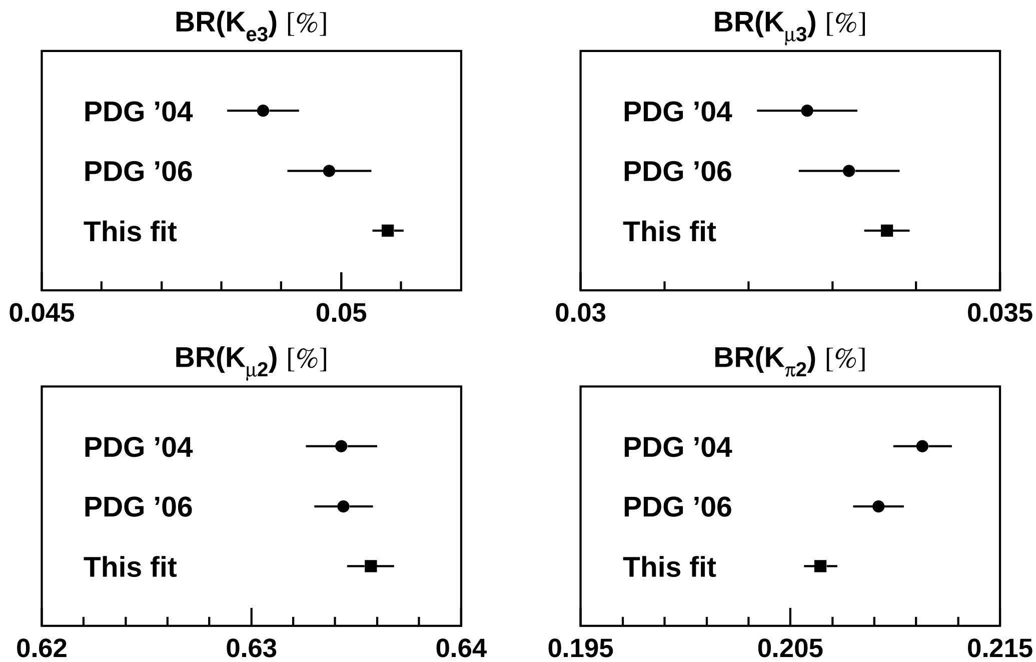

Figure 2: Evolution of average values for main $K^{\pm}$ BRs.

2.4 Measurement of BR(K_{e2})/BR($K_{\mu2}$)

Experimental knowledge of $K_{e2}/K_{\mu2}$ has been poor so far. The current world average of $R_K = \mathrm{BR}(K_{e2})/\mathrm{BR}(K_{\mu2}) = (2.45 \pm 0.11) \times 10^{-5}$ dates back to three experiments of the 1970s [26] and has a precision of about 5%. The three new preliminary measurements were reported by NA48/2 and KLOE (see Tab. 4): A preliminary result of NA48/2, based on about 4000 K_{e2} events from the 2003 data set, was presented in 2005 [27]. Another preliminary result, based on also about 4000 events, recorded in a minimum bias run period in 2004, was shown at KAON07[28]. Both results have independent statistics and are also independent in the systematic uncertainties, as the systematics are either of statistical nature (as e.g. trigger efficiencies) or determined in an independent way. Another preliminary result, based on about 8000 K_{e2} events, was presented at KAON07 by the KLOE collaboration [29]. Both, the KLOE and the NA48/2 measurements are inclusive with respect to the final state radiation bremsstrahlung contribution. The small contribution of $K_{l2\gamma}$ events from direct photon emission from the decay vertex was subtracted by each of the experiments. Combining these new results with the current PDG value yields a current world average of

$$R_K = (2.457 \pm 0.032) \times 10^{-5}, \tag{2.1}$$

in very good agreement with the SM expectation and, with a relative error of 1.3%, a factor three more precise than the previous world average.

	R_K [10^{-5}]
PDG 2006 [26]	2.45 ± 0.11
NA48/2 prel. ('03) [27]	$2.416 \pm 0.043 \pm 0.024$
NA48/2 prel. ('04) [28]	$2.455 \pm 0.045 \pm 0.041$
KLOE prel. [29]	$2.55 \pm 0.05 \pm 0.05$
SM prediction	2.477 ± 0.001

Table 4: Results and prediction for $R_K = \mathrm{BR}(K_{e2})/\mathrm{BR}(K_{\mu 2})$.

2.5 Measurements of $K_{\ell 3}$ slopes

2.5.1 Vector form factor slopes from $K_{\ell 3}$

For K_{e3} decays, recent measurements of the quadratic slope parameters of the vector form factor $(\lambda'_+, \lambda''_+)$ are available from KTeV [30], KLOE [31], ISTRA+ [32], and NA48 [33].

We show the results of a fit to the K_L and K^- data in the first column of Table 5, and only the K_L data in the second column. With correlations correctly taken into account, both fits give good values of χ^2/ndf. The significance of the quadratic term is 4.2σ from the fit to all data, and 3.5σ from the fit to K_L data only.

	K_L and K^- data 4 measurements $\chi^2/\mathrm{ndf} = 5.3/6$ (51%)	K_L data only 3 measurements $\chi^2/\mathrm{ndf} = 4.7/4$ (32%)
$\lambda'_+ \times 10^3$	25.15 ± 0.87	24.90 ± 1.13
$\lambda''_+ \times 10^3$	1.57 ± 0.38	1.62 ± 0.46
$\rho(\lambda'_+, \lambda+'')$	-0.941	-0.951
$I(K^0_{e3})$	0.154651(236)	0.154560(307)
$I(K^{\pm}_{e3})$	0.159005(241)	0.158912(315)

Table 5: Average of quadratic fit results for K_{e3} slopes

Including or excluding the K^- slopes has little impact on the values of λ'_+ and λ''_+; in particular, the values of the phase-space integrals change by just 0.07%.

KLOE, KTeV, and NA48 also quote the values shown in Table 6 for M_V from pole fits to $K_{L\,e3}$ data. The average value of M_V from all three experiments is $M_V = 875.3 \pm 5.4$ MeV with $\chi^2/\mathrm{ndf} = 1.80/2$. The three values are quite compatible with each other and reasonably close to the known value of the $K^{\pm *}(892)$ mass (891.66 ± 0.26 MeV). The values for λ'_+ and λ''_+ from expansion of the pole parameterization are qualitatively in agreement with the average of the quadratic fit results. More importantly, for the evaluation of the phase-space integrals, using the average of quadratic or pole fit results gives values of $I(K^0_{e3})$ that differ by just 0.03%. No additional error needs be assigned to account for differences obtained with quadratic and pole parameterizations for the vector form-factor slope.

Experiment	M_V (MeV)	$\langle M_V \rangle = 875.3 \pm 5.4$ MeV
KLOE	$870 \pm 6 \pm 7$	$\chi^2/\mathrm{ndf} = 1.80/2$
KTeV	881.03 ± 7.11	$\lambda'_+ \times 10^3 = 25.42(31)$
NA48	859 ± 18	$\lambda''_+ = 2 \times \lambda'^2_+$
		$I(K^0_{e3}) = 0.154695(192)$

Table 6: Pole fit results for K^0_{e3} slopes

2.5.2 Scalar and Vector form factor slopes from $K_{\ell 3}$

For $K_{\mu 3}$ decays, recent measurements of the slope parameters $(\lambda'_+, \lambda''_+, \lambda_0)$ are available from KTeV [30], KLOE [34], ISTRA+ [35], and NA48 [36]. Note that it is not possible, because of correlations, deduce the presence of quadratic term in $\tilde{f}_0(t)$ from the decay spectra. For the same reason fits with a linear parametrization give a wrong result for the slope λ_0.

Figure 3 shows the 1-σ contours from all the experimantal results (K_{e3} and $K_{\mu 3}$). It is immediately clear from the figure that the new NA48 results are difficult to accommodate. Performing the combinaton with and without the NA48 results for the $K_{\mu 3}$ form-factor slopes included we obtain fit probability values of 1×10^{-6} and 22.3% respectively. The results of the combination are listed in Table 7.

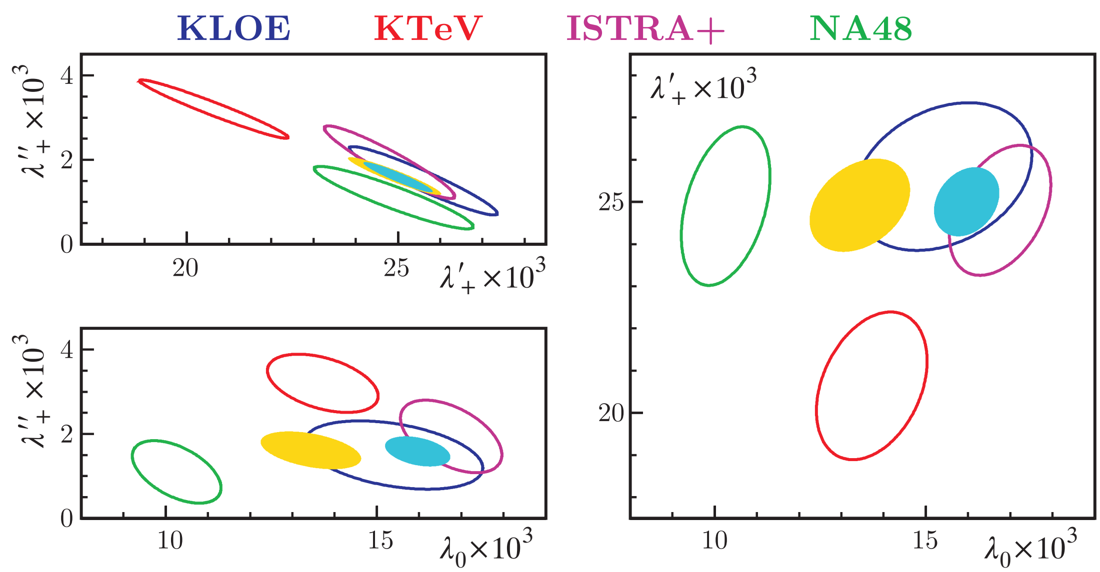

Figure 3: 1-σ contours for λ'_+, λ''_+, λ_0 determination from ISTRA+(pink ellipse), KLOE(blue ellipse), KTeV(red ellipse), NA48(green ellipse), and world average with(filled yellow ellipse) and without(filled cyan ellipse) the NA48 $K_{\mu 3}$ result.

The value of χ^2/ndf for all measurements is terrible; we are forced to quote the results with scaled errors. This leads to errors on the phase-space integrals that are ∼60% larger after inclusion of the new $K_{\mu 3}$ NA48 data.

	K_L and K^-	K_L only
Measurements	16	11
χ^2/ndf	54/13 (7×10^{-7})	33/8 (8×10^{-5})
$\lambda'_+ \times 10^3$	24.920 ± 1.105 ($S = 1.4$)	24.011 ± 1.544 ($S = 1.5$)
$\lambda''_+ \times 10^3$	1.612 ± 0.447 ($S = 1.3$)	1.974 ± 0.622 ($S = 1.6$)
$\lambda_0 \times 10^3$	13.438 ± 1.19 ($S = 1.9$)	11.682 ± 1.238 ($S = 1.7$)
$\rho(\lambda'_+, \lambda''_+)$	-0.944	-0.966
$\rho(\lambda'_+, \lambda_0)$	$+0.328$	$+0.715$
$\rho(\lambda''_+, \lambda_0)$	-0.439	-0.695
$I(K^0_{e3})$	0.154566(292)	0.154354(389)
$I(K^\pm_{e3})$	0.158918(300)	0.158700(400)
$I(K^0_{\mu3})$	0.102123(312)	0.101643(424)
$I(K^\pm_{\mu3})$	0.105073(321)	0.104578(437)
$\rho(I_{e3}, I_{\mu3})$	$+0.63$	$+0.89$

Table 7: Averages of quadratic fit results for K_{e3} and $K_{\mu3}$ slopes.

We have checked to see if the NA48 $K_{\mu3}$ data might show good consistency with the results of some other experiment in a less inclusive average. Fitting to only the $K_{\mu3}$ results from KTeV, NA48, and ISTRA+ gives $\chi^2/\text{ndf} = 27.5/6$ (0.01%). Fitting to only the $K_{L\,\mu3}$ results from KTeV, NA48 gives $\chi^2/\text{ndf} = 11.6/3$ (0.89%). The consistency of the NA48 data with these other measurements appears to be poor in any case.

The evaluations of the phase-space integrals for all four modes are listed in each case. Correlations are fully accounted for, both in the fits and in the evaluation of the integrals. The correlation matrices for the integrals are of the form

$$\begin{matrix} +1 & +1 & \rho & \rho \\ +1 & +1 & \rho & \rho \\ \rho & \rho & +1 & +1 \\ \rho & \rho & +1 & +1 \end{matrix}$$

where the order of the rows and columns is K^0_{e3}, $K^\pm_{e3}$, $K^0_{\mu3}$, $K^\pm_{\mu3}$, and $\rho = \rho(I_{e3}, I_{\mu3})$ as listed in the table.

Adding the $K_{\mu3}$ data to the fit does not cause drastic changes to the values of the phase-space integrals for the K_{e3} modes: the values for $I(K^0_{e3})$ and $I(K^\pm_{e3})$ in Table 7 are qualitatively in agreement with those in Table 5. As in the case of the fits to the K_{e3} data only, the significance of the quadratic term in the vector form factor is strong (3.6σ from the fit to all data).

3. Physics Results

3.1 Determination of $f_+(0)V_{us}$ and $V_{us}/V_{ud} \times f_K/f_\pi$

This section describe the results that are independent on the theoretical parameters $f_+(0)$ and f_K/f_π.

3.1.1 Determination of $f_+(0)V_{us}$

The value of $f_+(0)V_{us}$ has been determined from 1.5 using the world average values reported in 2 for lifetime, branching ratios and phase space integrals, and the radiative and SU(2) breaking corrections discussed in section 1.1.

The results are given in Table 8, and are shown in Fig. 4 for $K_L \to \pi e\nu$, $K_L \to \pi\mu\nu$, $K_S \to \pi e\nu$, $K^\pm \to \pi e\nu$, $K^\pm \to \pi\mu\nu$, and for the combination.

mode	$f_+(0)V_{us}$	% err	BR	τ	Δ	Int
$K_L \to \pi e\nu$	0.21625(60)	0.28	0.09	0.19	0.15	0.09
$K_L \to \pi\mu\nu$	0.21675(66)	0.31	0.10	0.18	0.15	0.15
$K_S \to \pi e\nu$	0.21542(134)	0.67	0.65	0.03	0.15	0.09
$K^\pm \to \pi e\nu$	0.21728(84)	0.39	0.26	0.09	0.26	0.09
$K^\pm \to \pi\mu\nu$	0.21758(111)	0.51	0.40	0.09	0.26	0.15
average	0.21661(46)					

Table 8: Summary of $f_+(0)V_{us}$ determination from all channels.

The average, $|V_{us}|f_+(0) = 0.21661(46)$, has an uncertainty of about of 0.2%. The results from the five modes are in good agreement, the fit probability is 58%. In particular, comparing the values of $f_+(0)V_{us}$ obtained from $K^0_{\ell 3}$ and $K^\pm_{\ell 3}$ we obtain a value of the SU(2) breaking correction

$$\delta^K_{SU(2)_{exp.}} = 2.86(39)\%$$

in agreement with the CHPT calculation reported in table 1 $\delta^K_{SU(2)} = 2.36(22)\%$.

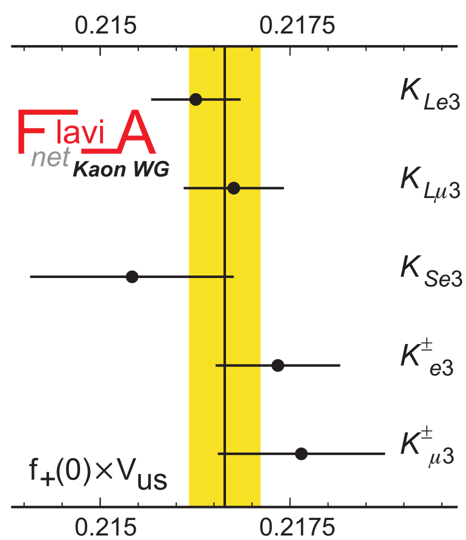

Figure 4: Display of $f_+(0)V_{us}$ for all channels.

3.1.2 Determination of $V_{us}/V_{ud} \times f_K/f_\pi$

Another determination of $|V_{us}|$ is obtained from $K_{\ell 2}$ decays. The most important mode is $K^+ \to \mu^+\nu$ which has been recently updated by KLOE, so that the relative uncertainty is now about 0.3%. To reduce hadronic uncertainties, in eq. (1.6) we have introduced the ratio $\Gamma(K^+ \to \mu^+\nu)/\Gamma(\pi^+ \to \mu^+\nu)$.

Using the world average values of BR$(K^\pm \to \mu^\pm\nu)$ and of $\tau^\pm$ given in section2 and the value of $\Gamma(\pi^\pm \to \mu^\pm\nu) = 38.408(7)\mu s^{-1}$ from [26] we obtain:

$$V_{us}/V_{ud} \times f_K/f_\pi = 0.27599 \pm 0.00059$$

3.2 The parameters f_+ and f_K/f_π

For the time being, such a highly precise measurement could not be translated to a similar error on the $|V_{us}|$ determination and therefore to the test of CKM unitarity. The obstacle

is obviously the difficulty to keep the theoretical uncertainties in $f_+(0)$ and f_K/f_π at the per-mil level.

3.2.1 $f_+(0)$ determination

In eq. (1.5), $f_+(0)$ is defined in the absence of electromagnetic corrections and I-spin breaking effects. While QCD is flavor blind the mass differences $m_u \neq m_d \neq m_s$ results in $f_+(0)$ being different from unity and also being different for charged and neutral kaons. In the following, by common convention, $f_+(0)$ is refers to neutral kaons, the difference for charged kaons is accounted for in the term $\delta^K_{SU(2)}$ of Eq. (1.5). $f_+(0)$ is calculable in non-perturbative QCD. In the flavor SU(3) limit, CVC ensures $f_+(0)$=1, but then $m_K = m_\pi$ and there are no weak decays. We can write

$$f_+(0) = 1 + f_2 + f_4 + \ldots \tag{3.1}$$

In chiral perturbation theory (ChPT) f_2 and f_4 stand for the leading and next-to-leading chiral corrections. The Ademollo–Gatto theorem ensures that terms $\propto (m_s - m_u)$ are absent for vector transitions and $f_2 = -0.023$ is unambiguously predicted in ChPT. The calculation of the chiral loop contribution, $\Delta(\mu)$ in

$$f_4 = \Delta(\mu) + f_4|^{\rm loc}(\mu)\,, \tag{3.2}$$

has been recently completed in ref. [37], but the full determination of f_4 necessitates an accurate estimation of the local counter-term $f_4|^{\rm loc}(\mu)$, which is $\mathcal{O}(p^6)$. Many theoretical

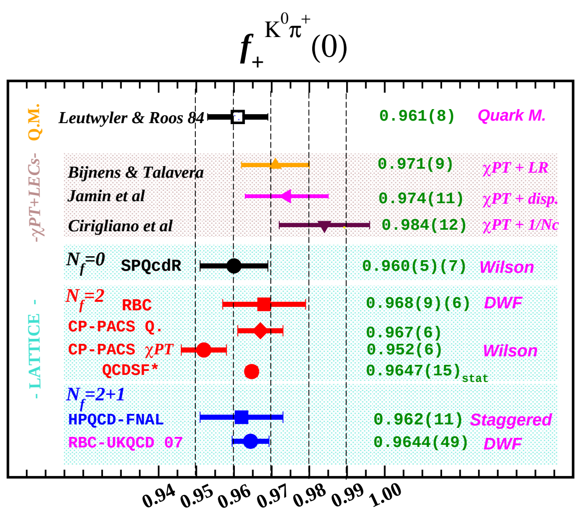

Figure 5: Present determinations of $f_+(0) \equiv f_+^{K^0\pi^-}(0)$ [38, 39, 40, 41] from lattice QCD and other approaches. Hints from ChPt are exploited from all these approaches.

approaches have been attempted over the years [41], essentially confirming the original estimate by Leutwyler and Roos which was obtained in a simple quark model [38]. The benefit of these new results, obtained using more sophisticated approaches, lies in the fact that we are nowadays in the position to control the systematic uncertainties of our calculations while with the quark models this is not possible. To stress the importance of

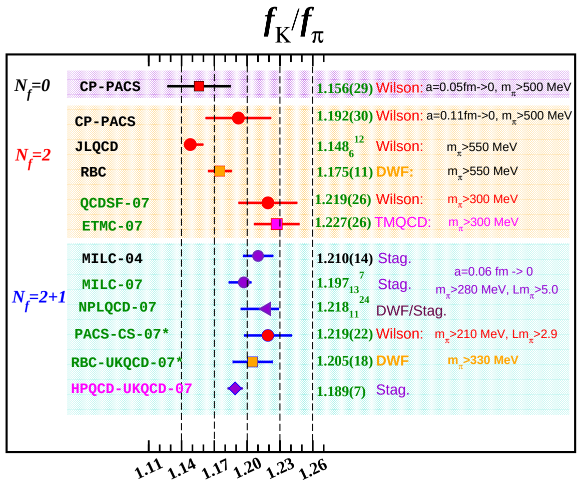

Figure 6: Summary of f_K/f_π estimates [42, 43, 44, 45]. All values are from Lattice QCD. In recent studies, sea quarks are getting light and data are matched to ChPt fits to determine the Low-energy-Constants (LEC).

the accurate determination of f_4, we should remind the reader that the experimental error on $|V_{us}|f_+(0)$ is only 0.2%, whereas the spread of theoretical estimates of $f_+(0)$ is still at the $1\% \div 2\%$ which is unsatisfactory.

Recent progress in lattice QCD gives us more optimism as far as the prospects of reducing the error on $f_+(0)$ to well below 1% are concerned [46]. Most of the currently available results obtained by using lattice QCD worked with the "heavy pions". Nevertheless, we can already see that the lattice QCD results are systematically lower than those obtained by the ChPT-inspired models. An important step to resolving this issue has been recently made by the UKQCD-RBC collaboration [39]. Their preliminary result $f_+(0) = 0.964(5)$ is obtained from the unquenched study with $N_F = 2+1$ flavors of the quarks which have a good chiral properties on the lattice with finite lattice spacing (so called, domain wall quarks), and their pions ($\gtrsim 300\,\mathrm{MeV}$) are much lighter than what is reported in the previous lattice QCD studies. Their overall error is estimated to be 0.5%, which is very encouraging. It is important to emphasize that they observe the chiral logarithmic corrections, those which appear in the form factor as f_2, which was never observed before (most probably because other studies were restrained to heavier pions). One should keep in mind, however, that present study is performed at a single value of the lattice spacing (i.e. $a = 0.12$ fm) and in a relatively small lattice box.

3.2.2 f_K/f_π determination

As we can see in eq. (1.6), the QCD uncertainty enters with

$$f_K/f_\pi = 1 + r_2 + \dots \tag{3.3}$$

In contrast to the semileptonic case discussed above, the Ademollo–Gatto theorem does not apply in this case and r_2 is not predicted unambiguously in ChPT. Rather one should fix the low energy constants from the lattice QCD studies of f_K/f_π [42]-[45]. Such obtained

values are summarized in fig. 6 from which we deduce that the present overall accuracy is about 1%. The novelty are the new lattice results with $N_F = 2 + 1$ dynamical quarks and pions as light as 280 MeV [42, 43], obtained by using the so-called staggered quarks, in which they covered a broad range of lattice spacings (i.e a=0.06 and 0.15 fm) and kept sufficiently large physical volumes (i.e. $m_\pi L \gtrsim 5.0$). It should be stressed, however, that the sensitivity of f_K/f_π to the lighter pions is larger than in the computation of $f_+(0)$ and that the chiral extrapolations are far more demanding in this case. Further improvement is expected soon. PACS-CS Collaboration [47] has recently presented preliminary results for $N_F = 2+1$ clover quarks with pion masses $\gtrsim 200$ MeV ($m_\pi L \gtrsim 2.9$ and $a = 0.09$ fm). For the moment, these simulations are on a single value of lattice volume and finite volume effects can be large.

3.2.3 A test of lattice calculation: the Callan-Treiman relation

As described in 1.2 the Callan-Treiman relation fixes the value of scalar FF at $t = \Delta_{K\pi}$ (the so-called Callan-Trieman point) to the ratio of the pseudoscalar decay constants f_K/f_π and the recent parametrization for the scalar FF[8] allows this constraint to be exploited. The net result is that the ratio $(f_K/f_\pi)/f_+(0)$ can be determined from the measurement of the scalar form factor.

Very recently KLOE [34] and NA48 [36] have presented results on the scalar form factor slopes using the dispersive parematrization. The older results of KTeV and ISTRA+ measurement of the scalar form factor slope performed using the 1st order Taylor expansion parametrization can be translated in the dispersive parametrization. The results are given in Table 9 for all 4 experiments in the case of pole parametrization for the vector form factor. The original KLOE and NA48 results are also shown for comparison.

Experiment	$\log(C)$ direct	$\log(C)^\dagger$
KTeV		0.203(15)
KLOE	0.207(24)	0.207(23)
NA48	0.144(14)	0.144(13)
ISTRA+		0.226(13)

† Estimated from λ_0 published.

Table 9: Experimental results of for log(C) and λ_+

Figure 7 shows the values for $f_+(0)$ determined from the scalar form factor slope measurements obtained using the Callan-Treiman relation and $f_K/f_\pi = 1.189(7)$. The value of $f_+(0) = 0.964(5)$ from UKQCD/RBC is also shown.

As already noticed in 2 the NA48 result is difficult to accommodate, and once compared with theory it violates the Fubini-Furlan theorem $f_+(0) < 1$. For this reason the NA48 result will be excluded when using the Callan-Treiman constraint.

The average of the experimental results on the FF's with the pole parametrization for

the vector FF and the dispersive paremetrization for the scalar form factor gives:

$$
\begin{aligned}
\lambda_+^c &= 0.0256 \pm 0.0002 \\
\lambda_0^c &= 0.0149 \pm 0.0007
\end{aligned} \tag{3.4}
$$

with correlation coefficient -0.32. The above result is then combined to the lattice determination of $f_K/f_\pi = 1.189(7)$ and $f_+(0) = 0.964(5)$ using the constraint given from the Callan-Treiman relation. The pole paremeterization has been used to describe the vector form factor. The results of the combination are given in table 10,

λ_+^c	λ_0^c	$f_+(0)$	f_K/f_π
0.02563(20)	0.01455(51)	0.963(44)	1.1913(61)
correlation matrix			
1.	-0.24	0.11	-0.13
	1.	-0.45	0.54
		1.	0.27
			1.

Table 10: Results from the form factor fit.

where $\log C = \lambda_0^c \Delta_{K\pi}/m_\pi^2 + 0.0398 \pm 0.0041$.

The fit probability is 39% confirming the agreement between experimental measurements and lattice determination. The accuracy of f_K/f_π is slightly improved. The improvement is better seen in the ratio $f_+(0)/(f_K/f_\pi)$, directly related to the Callan-Treiman constraint. This latter improvement is very effective in the constraining scalar currents.

3.3 Test of Cabibbo Universality or CKM unitarity

To test CKM unitarity we use the value $f_+(0)V_{us} = 0.21661(46)$ given in Table 8, $V_{us}/V_{ud} f_K/f_\pi = 0.27599(59)$ (see 3.1.2), $f_+(0) = 0.964(5)$, and $f_K/f_\pi = 1.189(7)$. The results are:

$$V_{us} = 0.22461 \pm 0.00124 \tag{3.5}$$

$$V_{us}/V_{ud} = 0.23211 \pm 0.00145 \tag{3.6}$$

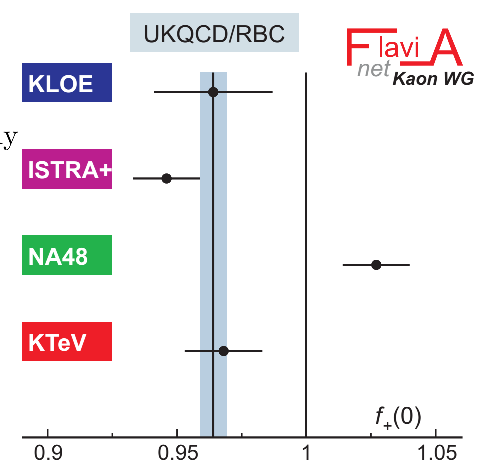

Figure 7: Values for $f_+(0)$ determined from the scalar form factor slope using the Callan-Treiman relation and $f_K/f_\pi = 1.189(7)$. The UKQCD/RBC result $f_+(0) = 0.964(5)$ is also shown.

These determinations can be used in a fit together with the the recent evaluation of $|V_{ud}|$ from $0^+ \to 0^+$ nuclear beta decays. The fit gives $|V_{ud}| = 0.97417(26)$ and $|V_{us}| = 0.2253(9)$, with $\chi^2/\mathrm{ndf} = 0.65/1$ (42%). The unitarity constraint can also be included, in which case the fit gives $V_{us} = \sin\theta_C = \lambda = 0.2255(7)$ and $\chi^2/\mathrm{ndf} = 0.80/2$ (67%). Both results are illustrated in Fig. 8.

As described in the introduction the test of CKM unitarity can be also interpreted as a test of universality of the lepton and quark gauge coupling:

$$G_{CKM}^2 = G_\mu^2 \text{ or } |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \text{ and } G_F \equiv G_\mu. \tag{3.7}$$

Using the results of the fit we obtain:

$$G_{CKM} = (1.16624 \pm 0.00039) \times 10^{-5} \text{ GeV}^{-2} \tag{3.8}$$

In perfect agreement with the value obtained from the measurement of the muon lifetime:

$$G_\mu = (1.166371 \pm 0.000007) \times 10^{-5} \text{ GeV}^{-2} \tag{3.9}$$

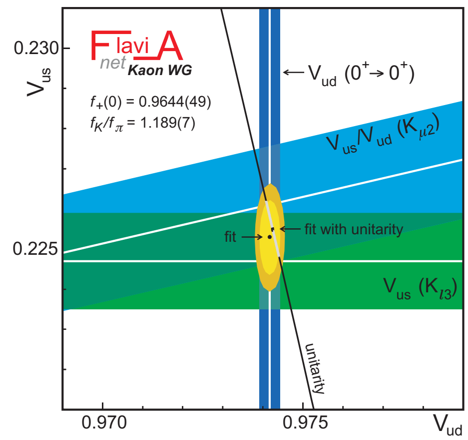

Figure 8: Results of fits to $|V_{ud}|$, $|V_{us}|$, and $|V_{us}|/|V_{ud}|$.

The current accuracy of the lepton-quark universality, sets important constraint to the model builders of the beyond SM physics scenarios. For example, the presence of a Z' (see Fig. 9, left) would affect the universality:

$$G_\mu = G_{CKM} \left[1 - 0.007 Q_{eL}(Q_{\mu L} - Q_{dL}) \frac{2 \ln(m_{Z'}/m_W)}{m_{Z'}^2/m_W^2 - 1} \right] \tag{3.10}$$

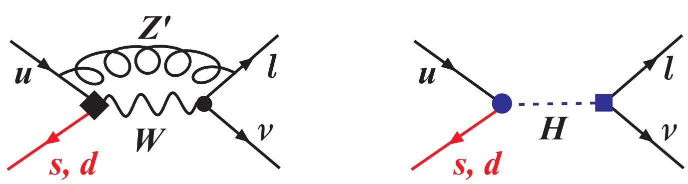

Figure 9: Z' and Higgs exchange.

In case of Z' in SO(10) grand unification theories, ($Q_{eL} = Q_{\mu L} = -3Q_{dL} = 1$) we obtain $m_{Z'} > 700$ GeV at 95% CL, to be compared the one set through the direct collider searches, $m_{Z'} > 720$ GeV [26].

3.3.1 Bounds from helicity suppressed processes

A particularly interesting observable is the ratio of the V_{us} values obtained from helicity suppressed processes to that obtained from helicity allowed modes: $R_{l23} = V_{us}(K_{\ell 2})/V_{us}(K_{\ell 3})$. According to[48, 8] the value of R_{l23} would be affected by the presence of scalar density or extra right-handed currents:

$$R_{l23} = 1 + \delta R_{l23} \tag{3.11}$$

To improve the accuracy of the determination of R_{l23} we use the the values of $f_+(0)$ and f_K/f_π obtained in 3.2.3 [1]. In addition, in this scenario both $0^+ \to 0^+$ nuclear beta decays

[1] R_{l23} depends only on the ratio $f_+(0)/(f_K/f_\pi)$

and $K_{\ell 3}$ are not affected and the unitarity constraint for these modes can be used. The fit described in the previous section has been performed assuming unitarity and allowing for two different values of V_{us} from helicity suppressed($K \to \mu\nu$) and allowed modes ($K \to \ell\pi\nu$, $0^+ \to 0^+$ nuclear beta decays). We obtain:

$$R_{l23} = 1.0028 \pm 0.0059 \tag{3.12}$$

If the scalar current is due to charged Higgs exchange as shown in Fig. 9, right:

$$\delta R_{l23} = (1 - tan^2\beta \frac{m^2_{K^\pm}/m^2_{H^\pm}}{1 + 0.01\tan\beta}) \tag{3.13}$$

and the measurement of R_{l23} can be used to set bounds on the charged Higgs mass and $\tan\beta$. Figure 10 shows the exlcuded region at 95% CL in the charged Higgs mass-tan β plane.

The measurement of BR($B \to \tau\nu$)[49] can be also used to set bound on the charged Higgs mass-$\tan\beta$ palne. While the $B \to \tau\nu$ can exclude quite an extensive region of this plane, there is an uncovered region in the exclusion corresponding to the change of sign of the correction. This region is fully covered by the $K \to \mu\nu$ and the positive correction solution for the $B \to \tau\nu$ is fully excluded.

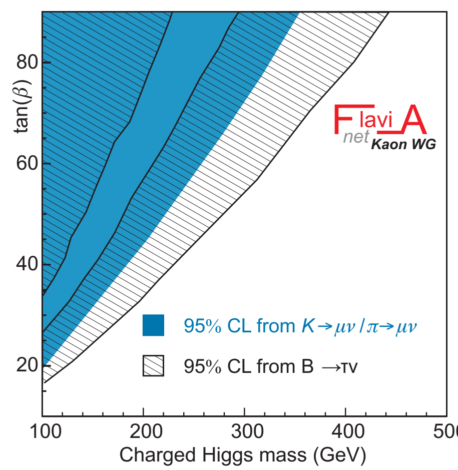

Figure 10: Exlucluded region in the charged Higgs mass-tan β plane. The region excluded by $B \to \tau\nu$ is also indicated.

3.4 Test of Lepton Flavour violation

3.4.1 Lepton universality and $K_{\ell 3}$ decays

Search for lepton flavour violation (LFV) in the semileptonic decays K_{e3} and $K_{\mu 3}$ is a test of the vector current of the weak interaction. It can therefore be compared to LFV tests in τ decays, but is different to LFV searches in π_{l2} and K_{l2} decays.

The results on the parameter $r_{\mu e} = R^{\rm Exp}_{K_{\mu 3}/K_{e3}}/R^{\rm SM}_{K_{\mu 3}/K_{e3}}$ is

$$r_{\mu e} = 1.0040 \pm 0.0044, \tag{3.14}$$

in excellent agreement with lepton universality. Furthermore, with a precision of 0.5% the test in K_{l3} decays has now reached the sensitivity of τ decays. The accuracy of this test slightly improves using FF's slope values of section3.2.3

$$r_{\mu e} = 0.9998 \pm 0.0040 \tag{3.15}$$

3.4.2 Lepton universality tests in $K_{\ell 2}$ decays

The ratio $R_K = \Gamma(K_{e2})/\Gamma(K_{\mu 2})$ can be precisely calculated within the Standard Model. Neglecting radiative corrections, it is given by

$$R_K^{(0)} = \frac{m_e^2}{m_\mu^2}\frac{(m_K^2 - m_e^2)^2}{(m_K^2 - m_\mu^2)^2} = 2.569 \times 10^{-5}, \tag{3.16}$$

and reflects the strong helicity suppression of the electronic channel. Radiative corrections have been computed within the model of vector meson dominance [5], yielding a corrected ratio of

$$R_K = R_K^{(0)}(1 + \delta R_K^{\text{rad.corr.}}) = 2.569 \times 10^{-5} \times (0.9622 \pm 0.0004) = (2.477 \pm 0.001) \times 10^{-5}. \tag{3.17}$$

Because of the helicity suppression of K_{e2} in the SM, the decay amplitude is a prominent candidate for possible sizeable contributions from new physics beyond the SM. Moreover, when normalizing to $K_{\mu 2}$ decays, it is one of the few kaon decays, for which the SM-rate is predicted with very high accuracy. Any significant experimental deviation from the prediction would immediately be evidence for new physics. However, this new physics would need to violate lepton universality to be visible in the ratio $K_{e2}/K_{\mu 2}$.

Recently it has been pointed out, that in a SUSY framework sizeable violations of lepton universality can be expected in K_{l2} decays [50]. At tree level, lepton flavour violating terms are forbidden in the MSSM. Loop diagrams, however, should induce lepton flavour violating Yukawa couplings as $H^+ \to l\nu_\tau$ to the charged Higgs boson H^+. Making use of this Yukawa coupling, the dominant non-SM contribution to R_K modify the ratio to

$$R_K^{\text{LFV}} \approx R_K^{\text{SM}} \left[1 + \left(\frac{m_K^4}{M_{H^\pm}^4}\right)\left(\frac{m_\tau^2}{M_e^2}\right)|\Delta_{13}|^2 \tan^6\beta\right]. \tag{3.18}$$

The lepton flavour violating term Δ_{13} should be of the order of $10^{-4} - 10^{-3}$, as expected from neutrino mixing. For moderately large $\tan\beta$ and $M_{H^\pm}$, SUSY contributions may therefore enhance R_K might by up to a few percent. Since the additional term in Eqn. 3.18 goes with the forth power of the meson mass, no similar effect is expected in π_{l2} decays.

The world average result for $R_K 2$ gives strong constraints for $\tan\beta$ and $M_{H^\pm}$ (Fig. 11 (left)). For a moderate value of $\Delta_{13} \approx 5 \times 10^{-4}$, $\tan\beta > 50$ is excluded for charged Higgs masses up to 1000 GeV/c^2 at 95% CL.

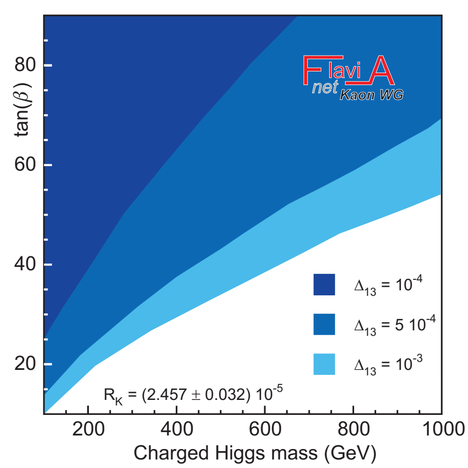

Figure 11: Exclusion limits at 95% CL on $\tan\beta$ and the charged Higgs mass $M_{H^\pm}$ from $|V_{us}|_{K\ell 2}/|V_{us}|_{K\ell 3}$ for different values of Δ_{13}.

4. Conclusions

Many new precise mesaurements about $K_{\ell 3}$ and $K_{\ell 2}$ decays properties have been performed recently allowing precise tests of the Standard Model to be performed. We determine:

$$f_+(0) \times V_{us} = 0.21661(46)$$

$$f_K/f_\pi \times V_{us}/V_{ud} = 0.27599(59)$$

using $f_K/f_\pi = 1.189(7)$ and $f_+(0) = 0.964(5)$ from recent lattice evaluation we obtain:

$$V_{us} = 0.2246(12) \tag{4.1}$$

$$V_{us}/V_{ud} = 0.2321(15) \tag{4.2}$$

in very good agreement with $|V_{ud}| = 0.97417(26)$ from $0^+ \to 0^+$ nuclear beta decays. These determinations can be used to evaluate the Fermi constant from the quark sector:

$$G_{CKM} = (1.16624 \pm 0.00039) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

to be compared with that obtained from the muone lifetime:

$$G_{\mu} = (1.166371 \pm 0.000007) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

The improved accuracy of these measurements sets non-trivial constraints for physics beyond the Standard Model. In particular, the comparison of the $|V_{us}|$ determinations from helicity suppressed and allowed processes sets a competitive lower bound on the charged higgs mass as a function of $\tan\beta$.

Very recentely the test of lepton flavour violation in helicity suppressed processes through the observable $R_K = \Gamma(K_{e2})/\Gamma(K_{\mu 2})$ as been improved significantly. The new world average

$$R_K = \mathrm{BR}(K_{e2})/\mathrm{BR}(K_{\mu 2}) = (2.45 \pm 0.11) \times 10^{-5}$$

gives the best constraint for the $e - \tau$ LFV coupling of MSSM.

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References

[1] FlaviaNet Kaon working group: `http://www.lnf.infn.it/wg/vus/`

[2] MuLan Collaboration, D.B. Chitwood et al., *Phys. Rev. Lett.* **99** (2007) 032001.

[3] J.C. Hardy and I.S. Towner, arXiv:0710.3181v1 [nucl-th].

[4] A. Sirlin, Nucl. Phys. B **196**, 83 (1982).

[5] W. J. Marciano, Phys. Rev. Lett. **93**, 231803 (2004) [arXiv:hep-ph/0402299]; V. Cirigliano and I. Rosell, JHEP **0710**, 005 (2007) [arXiv:0707.4464 [hep-ph]]; arXiv:0707.3439 [hep-ph].

[6] V. Cirigliano *et al.*, Eur. Phys. J. C **35** (2004) 53; Eur. Phys. J. C **23** (2002) 121; T. C. Andre, hep-ph/0406006.

[7] V. Cirigliano, H. Neufeld and I. Rosell, work in preparation.

[8] V. Bernard, M. Oertel, E. Passemar and J. Stern, arXiv:0707.4194 [hep-ph]; Phys. Lett. B **638**, 480 (2006) [arXiv:hep-ph/0603202].

[9] H. Leutwyler private comunication.

[10] T. Alexopoulos *et al.* [KTeV Collaboration], Phys. Rev. D **70**, 092006 (2004) [arXiv:hep-ex/0406002].

[11] A. Lai *et al.* [NA48 Collaboration], Phys. Lett. B **602**, 41 (2004) [arXiv:hep-ex/0410059].

[12] L. Litov [NA48 Collaboration], arXiv:hep-ex/0501048.

[13] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **626**, 15 (2005) [arXiv:hep-ex/0507088].

[14] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **632**, 43 (2006) [arXiv:hep-ex/0508027].

[15] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **638**, 140 (2006) [arXiv:hep-ex/0603041].

[16] A. Lai *et al.* [NA48 Collaboration], Phys. Lett. B **645**, 26 (2007) [arXiv:hep-ex/0611052].

[17] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **636**, 173 (2006) [arXiv:hep-ex/0601026].

[18] F. Ambrosino *et al.* [KLOE Collaboration], Eur. Phys. J. C **48**, 767 (2006) [arXiv:hep-ex/0601025].

[19] J. R. Batley *et al.*, Phys. Lett. B **653**, 145 (2007).

[20] J. R. Batley *et al.* [NA48/2 Collaboration], Eur. Phys. J. C **50**, 329 (2007) [arXiv:hep-ex/0702015].

[21] V. I. Romanovsky *et al.*, arXiv:0704.2052 [hep-ex].

[22] F. Ambrosino *et al.* [KLOE Collaboration], arXiv:0707.2532 [hep-ex].

[23] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **632**, 76 (2006) [arXiv:hep-ex/0509045].

[24] F. Ambrosino *et al.* [Kloe Collaboration], arXiv:0707.2654 [hep-ex].

[25] F. Ambrosino *et al.* [KLOE Collaboration], arXiv:0705.4408v3 [hep-ex].

[26] PDG, W.-M. Yao et al., *J. Phys.* **G33** (2006) 1.

[27] L. Fiorini, PoS **HEP2005**, 288 (2006);
L. Fiorini, ph.D. thesis, Pisa (2005);

[28] V. Kozhuharov, KAON07 International Conference.

[29] F. Ambrosino *et al.* [Kloe Collaboration], arXiv:0707.4623v1 [hep-ex].

[30] T. Alexopoulos *et al.* [KTeV Collaboration], Phys. Rev. D **70**, 092007 (2004) [arXiv:hep-ex/0406003].

[31] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **636**, 166 (2006) [arXiv:hep-ex/0601038].

[32] O. P. Yushchenko *et al.*, Phys. Lett. B **589**, 111 (2004) [arXiv:hep-ex/0404030].

[33] A. Lai *et al.* [NA48 Collaboration], Phys. Lett. B **604**, 1 (2004) [arXiv:hep-ex/0410065].

[34] F. Ambrosino *et al.* [KLOE Collaboration], arXiv:0707.4631 [hep-ex].

[35] O. P. Yushchenko *et al.*, Phys. Lett. B 581 (2004) 31.

[36] A. Lai *et al.* [NA48 Collaboration], Phys. Lett. B 647 (2007) 341.

[37] J. Bijnens and P. Talavera, Nucl. Phys. B **669**, 341 (2003); P. Post and K. Schilcher, Eur. Phys. J. C **25**, 427 (2002).

[38] H. Leutwyler and M. Roos, Z. Phys. C **25** (1984) 91.

[39] D. J. Antonio *et al.*, arXiv:hep-lat/0702026;

[40] M. Okamoto [Fermilab Lattice, MILC and HPQCD Collaborations], Int. J. Mod. Phys. A **20**, 3469 (2005);

[41] J. Portoles, arXiv:hep-ph/0703093; M. Jamin, J. A. Oller and A. Pich, JHEP **0402**, 047 (2004); V. Cirigliano, G. Ecker, M. Eidemuller, R. Kaiser, A. Pich and J. Portoles, JHEP **0504**, 006 (2005) [arXiv:hep-ph/0503108]; N. Tsutsui *et al.* [JLQCD Collaboration], PoS **LAT2005**, 357 (2006) [arXiv:hep-lat/0510068]; C. Dawson, T. Izubuchi, T. Kaneko, S. Sasaki and A. Soni, Phys. Rev. D **74**, 114502 (2006) [arXiv:hep-ph/0607162]; D. Becirevic *et al.*, Nucl. Phys. B **705**, 339 (2005) [arXiv:hep-ph/0403217]; D. Brommel *et al.*, arXiv:0710.2100 [hep-lat].

[42] C. Bernard *et al.*, arXiv:0710.1118 [hep-lat].

[43] E. Follana, C. T. H. Davies, G. P. Lepage and J. Shigemitsu [HPQCD Collaboration], arXiv:0706.1726 [hep-lat].

[44] C. Allton *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **76**, 014504 (2007) [arXiv:hep-lat/0701013].

[45] S. Aoki *et al.* [CP-PACS Collaboration], Phys. Rev. D **67**, 034503 (2003) [arXiv:hep-lat/0206009]; A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. D **65**, 054505 (2002) [Erratum-ibid. D **67**, 059901 (2003)] [arXiv:hep-lat/0105015]; S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. D **68**, 054502 (2003) [arXiv:hep-lat/0212039]; Y. Aoki *et al.*, Phys. Rev. D **72**, 114505 (2005) [arXiv:hep-lat/0411006]; M. Gockeler *et al.*, PoS **LAT2006**, 160 (2006) [arXiv:hep-lat/0610071]; C. Aubin *et al.* [MILC Collaboration], Phys. Rev. D **70**, 114501 (2004) [arXiv:hep-lat/0407028];S. R. Beane, P. F. Bedaque, K. Orginos and M. J. Savage, Phys. Rev. D **75**, 094501 (2007) [arXiv:hep-lat/0606023]; T. Ishikawa *et al.*, PoS **LAT2006**, 181 (2006) [arXiv:hep-lat/0610050]; B. Blossier *et al.* [European Twisted Mass Collaboration], arXiv:0709.4574 [hep-lat].

[46] Juettner, plenary talk at Lattice 2007; T. Kaneko, arXiv:0710.0698 [hep-ph].

[47] D. Kadoh *et al.* [CS Collaboration], arXiv:0710.3467 [hep-lat]; N. Ukita *et al.* [CS Collaboration], arXiv:0710.3462 [hep-lat].

[48] G. Isidori and P. Paradisi, Phys. Lett. B **639**, 499 (2006) [arXiv:hep-ph/0605012]; W. S. Hou, Phys. Rev. D **48**, 2342 (1993).

[49] K. Ikado *et al.*, Phys. Rev. Lett. **97**, 251802 (2006) [arXiv:hep-ex/0604018]; B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **76**, 052002 (2007) [arXiv:0708.2260 [hep-ex]].

[50] A. Masiero, P. Paradisi and R. Petronzio, hep-ph/0511289.